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An analysis of the heat and momentum transfer during rapid quenching of some microcrystalline materials from the melt

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The process of crystalline ribbon formation by the single roller technique by rapid quenching from the melt has been studied with the aid of boundary layer theory. An easy calculation method has been proposed for the solution of heat and momentum transfer equations for the case in which the melt is rapidly quenched on to a substrate. The proposed method takes into account the viscosity change due to the temperature reduction, the heat resistance of the puddle–substrate area and the latent heat of fusion separation on the liquid–solid phase interface. The thicknesses of the ribbon calculated with the aid of the proposed method show good correlation with the experimental results. The quenching rates within the volume of the puddle for Al–Cu eutectic alloy are considered.

1. Introduction

In the past two decades much attention has been paid to materials obtained by rapid quenching from the melt, because of the importance of their possible application [1–8]. Among the various rapid solidification techniques, the single-roller chill-block casting methods of planar flow casting (PFC) and chill block melt spinning (CBMS), in which continuously supplied melt is cooled by a wheel rotating at a high revolution rate, are most common at present because of their relative simplicity and ability to produce a large amount of rapidly solidified material in the form of metallic ribbon. The most important application of these methods until now has been the production of amorphous metallic ribbons. Recently, however, considerable attention has been paid to rapidly quenched crystalline materials, since it has become evident that such thermal treatment applied to certain materials leads to an improvement of their properties.

In the case of rapidly solidified crystalline materials the melt–spun ribbons exhibit dimensional variations which can give the ribbon non-uniform properties. The development of mathematical models for description of the melt-spinning of crystalline materials can be very helpful for obtaining quantitative relationships between the technological parameters and the ribbon dimensions, the thickness t and the width w [6, 8–10].

The aim of this paper is to examine the processes which occur in ribbon formation by rapid quenching from the melt, and their influence on the geometry of the crystalline ribbon obtained.

2. Description of the process

In both PFC and CBMS processes, a liquid melt pool or puddle (Fig. 1) forms in the region where the liquid impinges on the wheel surface. In effect the puddle spreads to such a size that the resultant ribbon thickness and width (CBMS only) satisfy the mass balance equation. The main characteristic of this puddle is its length, l, and the analysis of the experimental data shows that the thickness of the ribbon, t, strongly depends on the length of the melt pool. Taking into account this circumstance the conclusion can be drawn that the puddle length l is one of the main technological characteristics, and that the basic ribbon-forming processes take place within the melt puddle volume.

As the liquid metal is brought into contact with the cool surface of the moving substrate, a situation of simultaneous heat and momentum transfer arises. Due to the adhesion forces, a thin layer of the material is dragged out from the puddle volume which consequently forms the crystalline ribbon. Taking into account that the process of rapid quenching from the melt is continuous and that part of the melt may crystallize inside the volume of the puddle, for the ribbon thickness it follows that

$$t = \frac{1}{V_S}\int_{\delta_S}^{\delta_M} V_x(l, y)\,\mathrm{d}y + \delta_S(l) \qquad (1)$$

where V_S is the surface velocity of the substrate, δ_S is the thickness of the layer crystallized in the puddle volume, δ_M is the thickness of the boundary layer induced from the moving substrate and $V_x(x, y)$ is the x-component of the liquid velocity as a function of x and y at $x = l$. Hence in order to obtain the relationship between the thickness t of the ribbon and the key parameters of the process it is necessary to know $V_x(x, y)$ and $\delta_S(x)$.

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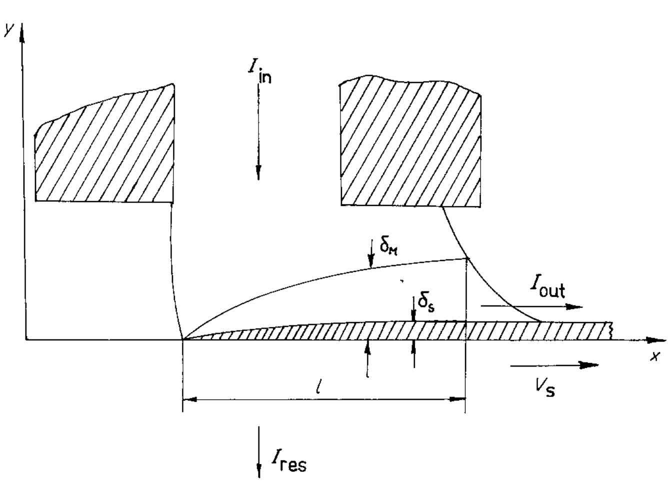

Figure 1 Schematic diagram of the molten alloy jet and puddle formed on the cold substrate. The thickness of the material crystallized in the puddle volume, δ_S, the thickness of the flow boundary layer, δ_M, the heat flows I_{in}, I_{out} and I_{res} and their directions are shown.

3. Equations, boundary conditions and assumptions

Mathematically the interaction of the heat and momentum transfer processes by rapid quenching from the melt, when a steady-state process is considered, is described from the Navier–Stokes and Fourier equations for a viscous incompressible liquid and the Fourier equation for heat transfer in the solid phase (the crystallized material) [11]:

$$
\begin{aligned}
(V\cdot\nabla)V &= -\frac{1}{\varrho^{L}}\nabla p + [\nabla\cdot(\nu\nabla)]V \qquad \nabla\cdot V = 0\\
V\cdot\nabla T &= D_T^{L}\nabla^2 T \qquad\qquad (2)\\
V_S\frac{\partial T}{\partial x} &= D_T^{S}\nabla^2 T
\end{aligned}
$$

where p is the pressure, ϱ^L and ν are the density and the kinematic viscosity of the melt, D_T^S and D_T^L are heat diffusion coefficients of the solid and liquid phases, V is the vector of the velocity, T is the temperature and ∇ and ∇^2 are Hamilton and Laplace operators, respectively.

This system of partial differential equations could be solved in explicit form only in some particular cases [11]. As the case discussed here cannot be reduced to one of them it is necessary to make some simplifications in order to obtain a solution of Equations 2.

In the present work the following assumptions are made, which are believed to be in agreement with experiment:

(a) The process is time-independent.

(b) Puddle width is constant, hence the flow can be treated as two-dimensional.

(c) The heat diffusion coefficients D_T^S and D_T^L are temperature-independent.

(d) The heat loss due to the radiation from the free surfaces of the puddle is negligible.

(e) The temperature of the puddle substrate contact area is a constant and equal to T_{SU} [6, 12].

(f) The process of momentum transfer is a laminar one.

(g) In the volume of the melt pool, due to the high velocity of the substrate surface ($\geqslant 10\,\text{m}\,\text{sec}^{-1}$) and the big difference between the melt and the substrate temperatures, V and T gradients obey the relationships $(\partial V/\partial x) \ll (\partial V/\partial y)$ and $(\partial T/\partial x) \ll (\partial T/\partial y)$.

The last assumption is the necessary condition for the application of boundary layer theory [13], which applied to Equations 2 and taking into account the above-listed conditions (a) to (f) is reduced to the following set of equations:

(a) For the liquid metal

$$V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} = \frac{\partial}{\partial y}\left(\nu\frac{\partial V_x}{\partial y}\right) \qquad (3a)$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \qquad (3b)$$

$$V_x\frac{\partial T}{\partial x} + V_y\frac{\partial T}{\partial y} = D_T^{L}\frac{\partial^2 T}{\partial y^2} \qquad (3c)$$

(b) For the solid metal

$$V_S\frac{\partial T}{\partial x} = D_T^{S}\frac{\partial^2 T}{\partial y^2} \qquad (3d)$$

with boundary conditions

$$
\begin{aligned}
&y \to \infty:\ V_x = 0,\ T = T_B\\
&y = \delta_S:\ V_x = V_S,\ V_y = 0,\ T = T_S,\\
&\qquad \lambda_L\frac{\partial T}{\partial y} - \lambda_S\frac{\partial T}{\partial y} = \varrho^L V_S L_m\frac{\mathrm{d}\delta_S}{\mathrm{d}x}\\
&y = 0:\ \lambda_S\frac{\partial T}{\partial y} = \alpha(T_{SU} - T_{SD})
\end{aligned}
$$

where T_B is the temperature of the supplied melt, T_S is the temperature of crystallization of the material, T_{SD} is the temperature of the substrate surface, α is the heat transfer coefficient at the substrate surface, λ_L and λ_S are the thermal conductivities of the liquid and solid phases, respectively, L_m is the latent heat of fusion and $\mathrm{d}\delta_S/\mathrm{d}x$ is the growth of the solid layer per unit length of the puddle [14].

At the present time there are two methods for integration of Equations 3. The first is the numerical explicit difference method [4, 5, 9, 10], and the second is through additional transformations to reduce the partial differential Equations 3 to an equation or a system of ordinary differential equations – the method of Blasius [13, 15].

Although it is very tempting to use the Blasius method for the solution of Equations 3, because of its mathematical simplicity and high accuracy, its application to the problem discussed gives rise to some difficulties, as the boundary conditions

$$\lambda_S \left(\frac{\partial T}{\partial y}\right)_{y=0} = \alpha(T_{SU} - T_{SD})$$

and

$$\lambda_L \left.\frac{\partial T}{\partial y}\right|_{y=\delta_S} - \lambda_S \left.\frac{\partial T}{\partial y}\right|_{y=\delta_S} = \varrho^L V_S L_m \frac{d\delta_S}{dx}$$

have to be transformed to ordinary differential equations too. If the boundary condition at the melt crystalline interface

$$\lambda_L \left.\frac{\partial T}{\partial y}\right|_{y=\delta_S} - \lambda_S \left.\frac{\partial T}{\partial y}\right|_{y=\delta_S} = \varrho^L V_S L_m \frac{d\delta_S}{dx}$$

can still be transformed from a partial to an ordinary differential equation (see Appendix A), for the boundary condition on the puddle–substrate contact area this is impossible. In order to avoid this difficulty in the present paper, a method is proposed based on the Blasius "similarity transformations" [15]:

$$V_x = \frac{\partial \psi}{\partial y};\ V_y = -\frac{\partial \psi}{\partial x};\ \xi = y\left(\frac{V_S}{xD_T^L}\right)^{1/2};$$
$$\psi = (xV_S D_T^L)^{1/2} f(\xi) \qquad (4)$$

where ψ is the stream function, ξ is a new independent variable and $f(\xi)$ is a function of it. This approach allows one to overcome the difficulties pointed out above. Application of the above transformations to Equations 3 reduces them to the following set of ordinary differential equations [7]:

$$(\nu f'')' + \frac{D_T^L}{2} ff'' = 0 \qquad (5a)$$

$$h'' + \frac{1}{2} fh' = 0 \qquad (5b)$$

$$h'' + \frac{1}{2}\frac{D_T^L}{D_T^S}\xi h' = 0 \qquad (5c)$$

with boundary conditions

$$\xi = 0:\ h = h_{SU}$$
$$\xi = \xi_S:\ f = \xi_S,\ f' = 1,\ h = 1,$$
$$h'_L = \frac{\lambda_S}{\lambda_L} h'_S + \frac{L_m \xi_S}{2C_p^L(T_B - T_S)}$$
$$\xi \to \infty:\ f' \to 0,\ h \to 0,\ h' \to 0$$

where h denotes the normalized temperature $h = (T_B - T)/(T_B - T_S)$, ξ_S is the profile of the crystallized material as a ξ variable and $h_{SU} = (T_B - T_{SU})/(T_B - T_S)$. The transformation of the melt–crystal interface boundary condition from x, y to ξ variables is given in detail in Appendix A. The mass balance Equation 1 is transformed into the expression

$$t = \left[\int_{\xi_S}^{\xi_M} f'(\xi)\,d\xi + \xi_S\right]\left(\frac{lD_T^L}{V_S}\right)^{1/2} \qquad (6)$$

In this process the relationships

$$\xi = y\left(\frac{V_S}{xD_T^L}\right),\quad V_x(x, y) = V_S f'(\xi)$$

obtained from Equation 4 have been used. Bearing in mind that

$$\int_{\xi_S}^{\xi_M} f'(\xi)\,d\xi = f(\xi_M) - f(\xi_S) = f(\infty) - \xi_S$$

where $f(\xi_M)$ and $f(\xi_S)$ are substituted by their equivalents [13], the ribbon thickness t is given by

$$t = f(\infty)\left(\frac{lD_T^L}{V_S}\right)^{1/2} \qquad (7)$$

Hence only the value of $f(\xi)$ at $\xi \to \infty$ is necessary in order to determine the thickness of the crystalline ribbon.

In our case Equations 5 are undetermined because of the required seven boundary conditions; there are only six available, as h_{SU} and ξ_S are unknown quantities. The only way to solve them in their present form is to assume that the value of h_{SU} is equal to h_{SD} (normalized temperature of the substrate). Unfortunately this case could not be realized by rapid quenching from the melt and it is interesting from the methodological point of view only [7]. To define the heat transfer resistance at the puddle–substrate contact area, an additional equation to Equations 5 is necessary which must be physically equivalent to the boundary condition

$$\lambda_S \left.\frac{\partial T}{\partial y}\right|_{y=\delta_S} = \alpha(T_{SU} - T_{SD})$$

Bearing in mind that this condition has the dimension $[W\,m^{-2}]$ (the heat flow through unit area) the balance of heat flow entering and leaving the volume of the puddle may be used in order to obtain the needed fourth equation. For convenience in the present work, instead of heat flow balance we use the power balance equation, as both of them are based on the energy conservation law. The heat flow entering the substrate $I_{res} = \alpha(T_{SU} - T_{SD})$, for example, is related to the power transmitted from the puddle to the substrate P_{res} by the equation

$$I_{res} = \frac{P_{res}}{ls} = \alpha(T_{SU} - T_{SD}) \qquad (8)$$

i.e. I_{res} and P_{res} are linearly dependent on each other because l and s in the case discussed are kept constant. Here s denotes the puddle width. According to the energy conservation law the power balance equation obeys the expression

$$P_{res} = P_{in} - P_{out} \qquad (9)$$

where P_{in} is the power transmitted to the system by means of the molten material entering the puddle volume and P_{out} is the power transmitted out from the puddle by means of the material, both liquid and solid, dragged out from the moving substrate.

The power transmitted to the puddle volume is expressed by the relationship

$$P_{in} = \varrho^L C_p^L V_i s T_B d \qquad (10)$$

where C_p^L is the specific heat capacity of the melt, d is the width of the melt flow entering the puddle and V_i is the velocity of this flow. According to the mass balance equation the following chain of equations is fulfilled:

$$V_i d = tV_S = V_S f(\infty)\left(\frac{lD_T^L}{V_S}\right)^{1/2} \tag{11}$$

where t is substituted by its equal from Equation 8. Substituting Equation 11 into Equation 10, for P_{in} one finally obtains

$$P_{in} = \varrho^L C_p^L T_B s\, V_S f(\infty)\left(\frac{lD_T^L}{V_S}\right)^{1/2} \tag{12}$$

The power P_{out} consists of two components: the power P_{out}^S transmitted out from the puddle with the solidified material, and P_{out}^L with the liquid layer:

$$P_{out} = P_{out}^L + P_{out}^S \tag{13}$$

Taking into account that

$$\begin{aligned} P_{out}^S &= s\varrho^S V_S \int_0^{\delta_S} C_p^S T(l, y)\, dy \\ &= s\varrho^S V_S C_p^S \int_0^{\delta_S} T(l, y)\, dy \end{aligned} \tag{14}$$

and that

$$\begin{aligned} P_{out}^L &= s\varrho^L \int_{\delta_S}^{\delta_M} C_p^L V_x(l, y)\, T(l, y)\, dy \\ &= s\varrho^L C_p^L \int_{\delta_S}^{\delta_M} V_x(l, y)\, T(l, y)\, dy \end{aligned} \tag{15}$$

thus the full power transmitted out from the puddle is given by

$$\begin{aligned} P_{out} = s\Big[& V_S C_p^S \varrho^S \int_0^{\delta_S} T(l, y)\, dy \\ & + C_p^L \varrho^L \int_{\delta_S}^{\delta_M} V_x(l, y) T(l, y)\, dy\Big] \end{aligned} \tag{16}$$

where in Equations 14 to 16 it is assumed that C_p^S and C_p^L are temperature-independent. As is seen, the value of P_{out} depends on the functions $V_x(l, y)$ and $T(l, y)$ ($x = l$). It can be shown that the change of the boundary condition on the liquid–solid interface changes the mode of $V_x(l, y)$ and $T(l, y)$, i.e. the values of these functions are strongly influenced by the processes within the puddle volume. Thus the heat separation on the liquid–solid interface in Equation 16 is accounted for by the mode of the functions $V_x(x, y)$ and $T(x, y)$ at $x = l$.

The transformation of the last equation from x, y to ξ variables yields the expression

$$\begin{aligned} P_{out} = sV_S\Big[& -(T_B - T_S)\Big(\varrho^S C_p^S \int_0^{\xi_S} h(\xi)\, d\xi \\ & + \varrho^L C_p^L \int_{\xi_S}^{\xi_M} h(\xi) f'(\xi)\, d\xi\Big) \\ & + T_B \xi_S(\varrho^S C_p^S - \varrho^L C_p^L) + \varrho^L C_p^L T_B f(\infty)\Big] \\ & \times \left(\frac{lD_T^L}{V_S}\right)^{1/2} \end{aligned} \tag{17}$$

where the equations

$$\int_0^{\xi_S} d\xi = \xi_S, \quad \int_{\xi_S}^{\xi_M} f'(\xi)\, d\xi = f(\infty) - \xi_S,$$

$$T = T_B - (T_B - T_S)h(\xi)$$

have been used in the process. Substituting Equations 12 and 17 into Equation 9 yields the expression

$$\begin{aligned} P_{res} = sV_S\Big[& (T_B - T_S)\Big(\varrho^S C_p^S \int_0^{\xi_S} h(\xi)\, d\xi \\ & + \varrho^L C_p^L \int_{\xi_S}^{\xi_M} h(\xi) f'(\xi)\, d\xi\Big) \\ & - T_B \xi_S(\varrho^S C_p^S - \varrho^L C_p^L)\Big]\left(\frac{lD_T^L}{V_S}\right)^{1/2} \end{aligned} \tag{18}$$

which when substituted into Equation 8 leads us to the fourth equation we were looking for:

$$\begin{aligned} \alpha(h_{SD} - h_{SU}) = \Big[& \varrho^S C_p^S \int_0^{\xi_S} h(\xi)\, d\xi \\ & + \varrho^L C_p^L \int_{\xi_S}^{\xi_M} f'(\xi) h(\xi)\, d\xi \\ & - \frac{T_B}{T_B - T_S}\xi_S\,(\varrho^S C_p^S - \varrho^L C_p^L)\Big] \\ & \times \left(\frac{V_S D_T^L}{l}\right)^{1/2} \end{aligned} \tag{19}$$

This equation together with Equations 5 forms a system of integral–differential equations:

$$(vf'')' + \frac{D_T^L}{2} ff'' = 0 \tag{20a}$$

$$h'' + \tfrac{1}{2} fh' = 0 \tag{20b}$$

$$h'' + \tfrac{1}{2}\frac{D_T^L}{D_T^S}\xi h' = 0 \tag{20c}$$

$$\begin{aligned} \alpha(h_{SD} - h_{SU}) = \Big[& \varrho^S C_p^S \int_0^{\xi_S} h(\xi)\, d\xi \\ & + \varrho^L C_p^L \int_{\xi_S}^{\xi_M} f'(\xi) h(\xi)\, d\xi \\ & - \frac{T_B}{T_B - T_S}\xi_S\,(\varrho^S C_p^S - \varrho^L C_p^L)\Big] \\ & \times \left(\frac{V_S D_T^L}{l}\right)^{1/2} \end{aligned} \tag{20d}$$

with boundary conditions

$$\xi = 0\colon h = h_{SU}$$

$$\xi = \xi_S\colon f = \xi_S, \quad f' = 1, \quad h = 1,$$

$$h'_L = \frac{\lambda_S}{\lambda_L} h'_S + \frac{L_m \xi_S}{2C_p(T_B - T_S)}$$

$$\xi \to \infty\colon f' \to 0, \quad h \to 0, \quad h' \to 0$$

which solves the problem formulated in the present paper. The integration of Equations 20 yields the values of $f'(\xi)$ and $h(\xi)$, together with the velocity and the temperatures of the liquid and solid phases:

$$V_x = V_S f'(\xi) \qquad V_y = \tfrac{1}{2}\left(\frac{V_S D_T^L}{x}\right)^{1/2}$$

$$\times\ (\xi f'(\xi) - f(\xi))$$

$$T = T_B - (T_B - T_S)h(\xi)$$

The integration of Equations 20 gives also the value of

TABLE I Nomenclature and selected materials properties

Nomenclature		Material			
		Al	Cu	Fe	Al–Cu eutectic
Heat capacity ($\text{kJ kg}^{-1}\text{K}^{-}$),	C_p^{L}	1.08	0.495	0.456	0.779
	C_p^{S}	1.04	0.384	0.456	0.779
Density ($10^{-3}\,\text{kg m}^{-3}$),	ϱ^{L}	2.36	8.03	7.40	3.3
	ϱ^{S}	2.70	8.93	7.86	3.3
Thermal diffusion coefficient ($10^{5}\,\text{m}^2\,\text{sec}^{-1}$),	D_T^{L}	3.61	4.17	0.966	3.45
	D_T^{S}	7.44	11.6	1.02	3.45
Latent heat of fusion ($10^{-5}\,\text{J kg}^{-1}$),	L_{m}	3.90	2.39	2.72	3.42
Viscosity ($10^{7}\,\text{m}^2\,\text{sec}^{-1}$),	v_0	1.52	0.881	0.367	See Equation 28
Temperature of crystallization (K),	T_{S}	933	1356	1811	821
Temperature of the melt (K),	T_{B}	1053	1476	1911	941
Activation energy (kJ mol^{-1}),	E	9.34	17.0	46	See Equation 28
Thermal conductivity ($\text{W m}^{-1}\text{K}^{-1}$),	λ_{L}	92	165.6	32.6	88.8
	λ_{S}	209	397	36.7	88.8
Heat transfer coefficient ($10^{-5}\,\text{W m}^{-2}\text{K}^{-1}$)	α	6	6	3	6
Temperature of the substrate (K),	T_{SD}	298	298	298	298

$f(\xi)$ at infinity, $f(\infty)$, which according to Equation 7 defines the thickness of the crystalline ribbons.

One of the most important characteristics of the process of rapid quenching from the melt is the value of the cooling rate during the process of ribbon formation. Mathematically, this characteristic is given by the expression $\dot{T} = \mathrm{d}T/\mathrm{d}\tau$, which takes the form

$$\dot{T} = \frac{\mathrm{d}T}{\mathrm{d}\tau} = \frac{\partial T}{\partial \tau} + V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} = V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} \qquad (21)$$

since the temperature is a function of x and y.

The Blasius transformations (Equations 4) lead to the following expressions for the cooling rate in the puddle volume: for the liquid

$$|\dot{T}|^{\text{L}} = \frac{1}{2x} V_{\text{S}}(T_{\text{B}} - T_{\text{S}})h'(\xi)f(\xi) \qquad (22)$$

and for the solid phase

$$|\dot{T}|^{\text{S}} = \frac{1}{2x} V_{\text{S}}(T_{\text{B}} - T_{\text{S}})\xi h'(\xi) \qquad (23)$$

With the aid of Equations 22 and 23 the values of the quenching rate in every point of the puddle volume can be calculated, if the values of $f(\xi)$ and $h(\xi)$ are known.

Except for Equation 20c, the system of equations so obtained cannot be solved analytically (see Appendix B), and for this purpose an algorithm has been created [12] which allows the Equations 20 to be solved numerically.

4. Testing of the method

4.1. On the dependences $t = t(\tau_1)$ for aluminium, copper and Al_2Cu–Al eutectic alloy

To test the method described in the present work three materials have been used, namely pure aluminium and copper and the eutectic alloy Al_2Cu–Al, all of them subjects of both theoretical and experimental interest [6, 8, 16–18].

All constants used in the computational process are listed in Table I. It is assumed that all the constants are temperature-independent, except for the kinematic viscosity of the melt, which obeys the following temperature dependence [19]:

$$v = v_0 \exp{(E/RT)} \qquad (24)$$

To calculate the thickness of crystalline ribbon for the case of pure aluminium, the empirical relationship for the dwell time τ_1 ($\tau_1 = l/V_{\text{S}}$), obtained from Katgerman and Van den Brink [8], has been used:

$$\tau_1 = 8.61 \times 10^{-3}\, V_{\text{S}}^{-1.42}\,\text{sec} \qquad (25)$$

The values for the crystalline ribbon thickness, calculated by the method presented here, are compared in Table II with the experimentally obtained data from the same authors. As we can see, all calculated values are within the error limits of the experimental data. The only exception is for $V_{\text{S}} = 19.1\,\text{m sec}^{-1}$, which is approximately 10% bigger than the experimentally obtained thickness for this velocity.

The results calculated for the case of pure copper and aluminium are plotted in log t–log τ_1 coordinates in Fig. 2, from which technologically important dependences $t = t(\tau_1)$ are derived:

$$t_{\text{Cu}} = 1.86 \times 10^{-1}\tau_1^{0.85}\,\mu\text{m} \qquad (26)$$

and

$$t_{\text{Al}} = 2.88 \times 10^{-2}\tau_1^{0.647}\,\mu\text{m} \qquad (27)$$

Since, in both cases, in the process of calculation all technological parameters (α, T_{SD}, $T_{\text{B}} - T_{\text{S}}$ etc.) have been taken equal, the observed difference between the

TABLE II Values of the calculated and experimentally obtained thicknesses, as a function of V_{S}, for the case of pure aluminium [8]

No.	V_{S} (m sec^{-1})	$t_{\text{exp}} \times 10^6$ (m)*		$t_{\text{calc}} \times 10^6$ (m)
		t_{min}	t_{max}	
1	47.7	33	54	37.1
2	31.8	42	58	54.7
3	19.1	56	79	88.2
4	12.7	112.5	133	127.3
5	8.2	179	229	186.8

*t_{min} and t_{max} denote the limits of the experimental data dispersion [8].

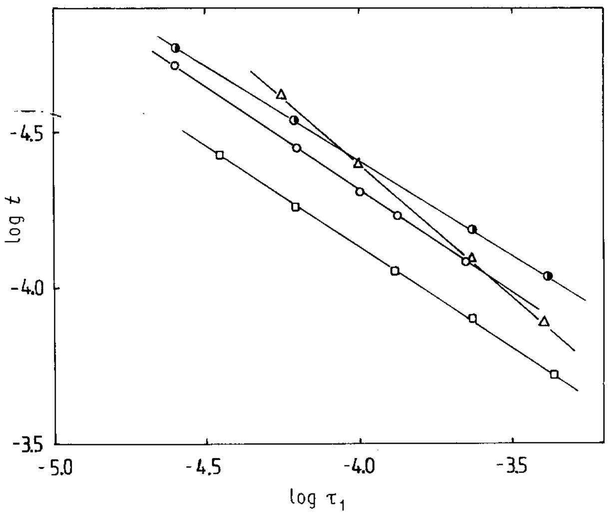

Figure 2 $\log t$ against $\log \tau_l$ plot for four different materials: (□) pure aluminium, (△) pure copper, (◑) pure iron and (○) Al–Cu eutectic alloy.

relationships $t = t(\tau_l)$ for pure aluminium and copper are obviously a consequence of the physical properties of the quenched materials. Hence the differences in the relationships $t = t(\tau_l)$ observed experimentally, for the different materials, are due to the significant role of the physical properties of the materials and also (and not only, as commonly believed) to the melt-substrate contact characteristics.

The kinematic viscosity for the eutectic alloy Al_2Cu–Al used in the computational process has been computed using the following equation [19]:

$$\ln \nu_{\text{Al-Cu}} = N_{\text{Al}} \ln \nu_{\text{Al}} + N_{\text{Cu}} \ln \nu_{\text{Cu}} \qquad (28)$$

where $N_{\text{Al}} = 0.827$ and $N_{\text{Cu}} = 0.173$ are the atomic proportions of aluminium and copper in the eutectic. It is assumed, by the application of Equation 28, that the viscosity of the eutectic components in the melt have the same functional dependence as the melts of the pure components. The calculated thicknesses of the ribbon for different dwell times are plotted in $\log t$–$\log \tau_l$ coordinates on Fig. 2, from which the following theoretical dependence is derived:

$$t = 1.96 \times 10^{-2} \tau_l^{0.65} \, \mu\text{m} \qquad (29)$$

Figure 3 shows a cut along the x axis of the puddle for rapid quenching of Al–Cu eutectic alloy, for two different dwell times, $\tau_l = 2.25 \times 10^{-4}$ sec (Fig. 3a) and 10^{-4} sec (Fig. 3b). As can be seen, a decrease of the dwell time, the respective decrease of the puddle length at constant velocity ($\tau_l = l/V_S$), or an increase of the velocity for constant puddle length leads to an increase of the quenching rate within the puddle volume. The noticeable diminishment of the cooling rate near the bottom of the puddle is due to the latent heat of fusion release at the solid–liquid interface.

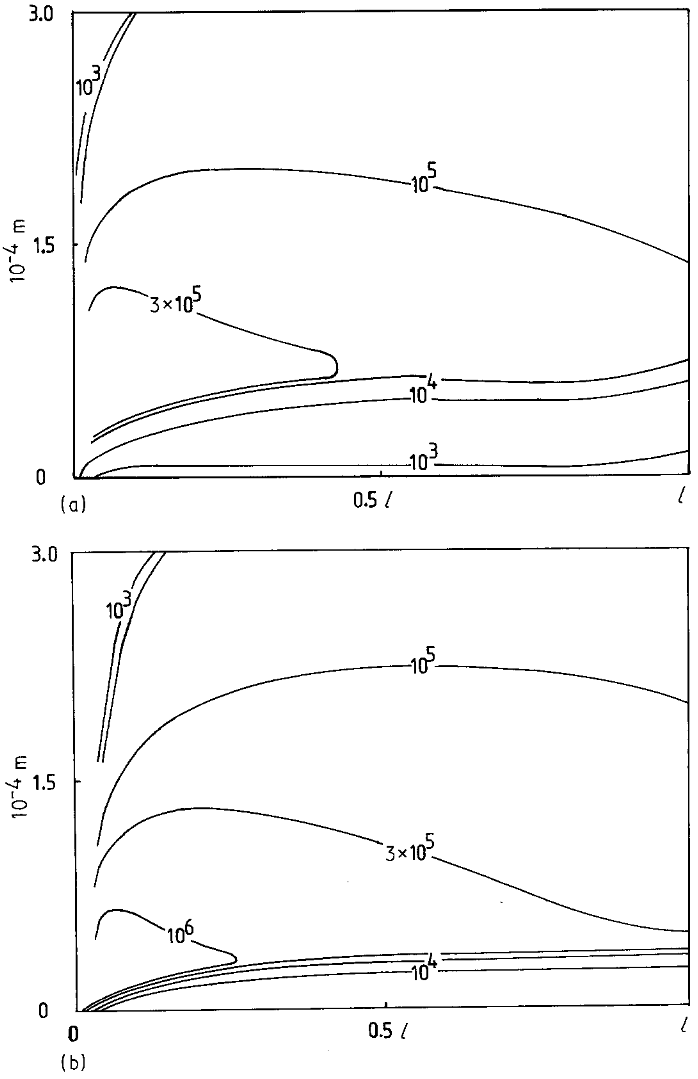

Figure 3 A cut along the x axis of the puddle, by rapid quenching of Al–Cu eutectic alloy, for two different dwell times: (a) $\tau_l = 2.25 \times 10^{-4}$ sec and (b) $\tau_l = 10^{-4}$ sec. The distribution of the quenching rates in the puddle volume is shown.

4.2. On the dependences $t = t(\tau_1)$ and $t = t(V_S)$ for pure iron

To test additionally the method described above, the dependences $t = t(\tau_1)$ and $t = t(V_S)$ for pure iron have been studied. A similar investigation of the dependence $t = t(V_S)$ has been carried out by Sun and Davies [9], who used for the integration of the heat and momentum transfer equations the finite difference method. This offers a good opportunity to compare the results obtained with the aid of the method discussed here with those obtained with the finite difference method.

Applying the equations from Section 3, and using the procedure described in Section 4.1, for the dependence $t = t(\tau_1)$ (see Fig. 2) one obtains

$$t = 10^{-2}\tau_1^{0.6} = 10^{-2}\left(\frac{l}{V_S}\right)^{0.6} \qquad (30)$$

where the data used for the computational process are listed in Table I. According to Davies and co-workers [9, 20], the dependence $l = l(V_S)$ is described by the empirical equation

$$l = 2.04 + \frac{22.4}{V_S}\ \mathrm{mm} \qquad (31)$$

The logarithm of the predicted values of t, obtained after substitution of Equation 31 into Equation 30, is plotted in Fig. 4 against the logarithm of the substrate velocity. Thus for the required relationship $t = t(V_S)$ one obtains

$$t_{\mathrm{pred}} = \frac{524.8}{V_S^{0.77}}\ \mu\mathrm{m} \qquad (32)$$

where V_S is in m sec^{-1}.

For the same dependence the finite difference method gives the following equation [9]:

$$t_{\mathrm{pred}} \propto V_S^{-A} = V_S^{-0.61} \qquad (33)$$

As is pointed out by the authors cited above, the value for A in this dependence differs from the experimentally obtained data, as for microcrystalline alloys A measures 0.67 to 0.83 [9]. The method discussed here gives for A the value of 0.77, which taking into account the above comments is in good agreement with experiment.

5. Conclusion

In the present paper the ribbon formation processes for microcrystalline materials by rapid quenching from the melt are studied. A method for solution of the heat and momentum transfer equations has been proposed, taking into account the existence of heat transfer resistance on the puddle–substrate contact area and the latent heat of fusion separation on the liquid–solid interface.

A comparison of the microcrystalline ribbon thicknesses for the case of pure aluminium, calculated with the aid of the Equations 20, shows a satisfactory agreement with the experimentally measured values. Taking this into account, a general conclusion can be drawn that the method presented here correctly describes the processes of ribbon formation by rapid quenching from the melt; it may be used for determination of the dependences $t = t(\tau_1)$ in the case of crystalline materials which are practical interest.

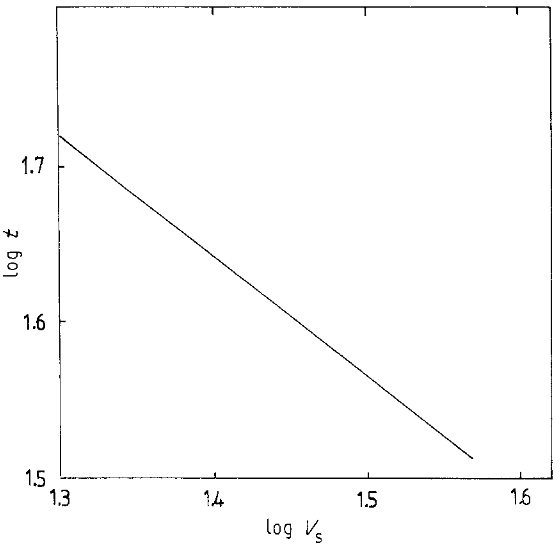

Figure 4 Predicted relationship of log t_{pred} against log V_S, indicating a power-law relation $t_{\mathrm{pred}} \propto V_S^{-A}$ with $A = 0.77$.

This method can be used successfully for investigation of the casting parameters as well, which cannot be measured by the standard methods, for example the heat transfer coefficient α.

In the paper some fields are given for the application of the method discussed, to the materials aluminium, copper, iron and Al–Cu eutectic alloy (see Figs 2; 3a, b; 4). A more detailed investigation of some alloys of practical interest, with the aid of the method proposed here, will be the subject of another paper.

Appendix A

When rapid quenching from the melt is applied to materials which undergo a liquid–solid phase transition, latent heat of freezing is generated at the liquid–solid interface. Mathematically, this circumstance is described [14] by the equation

$$\lambda_S \left.\frac{\partial T}{\partial y}\right|_{y=\delta_S} - \lambda_L \left.\frac{\partial T}{\partial y}\right|_{y=\delta_S} = \varrho^L V_S L_m \frac{\mathrm{d}\delta_S}{\mathrm{d}x} \qquad (\mathrm{A1})$$

The possible application of the Blasius method to the heat and momentum transfer equations for the case discussed depends on whether the boundary condition (Equation A1) may be transformed to an ordinary differential equation with argument ξ, or not. Here, a detailed description of the process of the transformation of Equation A1 from (x, y) to ξ variables is given.

The substitution of x and y with ξ in $\partial T/\partial y$ yields the expression

$$\frac{\partial T}{\partial y} = -(T_B - T_S)\frac{\partial h}{\partial y} = -(T_B - T_S)\frac{\mathrm{d}h}{\mathrm{d}\xi}\frac{\partial \xi}{\partial y}$$
$$= -(T_B - T_S)\frac{\mathrm{d}h}{\mathrm{d}\xi}\left(\frac{V_S}{xD_T^L}\right)^{1/2} \qquad (\mathrm{A2})$$

On the other hand, since $\xi_S = \delta_S(V_S/xD_T^L)^{1/2}$, the derivative $\mathrm{d}\delta_S/\mathrm{d}x$ yields

$$\frac{\mathrm{d}\delta_S}{\mathrm{d}x} = \frac{\xi_S}{2}\left(\frac{D_T^L}{xV_S}\right)^{1/2} \qquad (\mathrm{A3})$$

as ξ_S is a constant. Substituting the last two expressions into Equation A1, the following relationship is obtained:

$$\left.\frac{\mathrm{d}h}{\mathrm{d}\xi}\right|^{\mathrm{L}}_{\xi=\xi_{\mathrm{S}}} - \frac{\lambda_{\mathrm{S}}}{\lambda_{\mathrm{L}}}\left.\frac{\mathrm{d}h}{\mathrm{d}\xi}\right|^{\mathrm{S}}_{\xi=\xi_{\mathrm{S}}} = \frac{\varrho^{\mathrm{L}} L_{\mathrm{m}} D^{\mathrm{L}}_{T}}{2(T_{\mathrm{B}} - T_{\mathrm{S}})\lambda_{\mathrm{L}}}\,\xi_{\mathrm{S}}$$

$$= \frac{L_{\mathrm{m}}}{2(T_{\mathrm{B}} - T_{\mathrm{S}})C^{\mathrm{L}}_{p}}\,\xi_{\mathrm{S}} \qquad \text{(A4)}$$

which is the boundary condition (of Equation A1) transformed from (x, y) to ξ variables.

Appendix B

Analytically, Equation 20c is solved by separation of the variables, which yields for $h(\xi)$ and $h'(\xi)$ the expressions

$$h'(\xi) = h'(0)\exp\left(-\frac{1}{4}\frac{D^{\mathrm{L}}_{T}}{D^{\mathrm{S}}_{T}}\,\xi^{2}\right) \qquad \text{(B1)}$$

$$h(\xi) - h(0) = h'(0)\int_{0}^{\xi}\exp\left(-\frac{1}{4}\frac{D^{\mathrm{L}}_{T}}{D^{\mathrm{S}}_{T}}\,\eta^{2}\right)\mathrm{d}\eta$$

$$= h'(0)\left(\pi\frac{D^{\mathrm{S}}_{T}}{D^{\mathrm{L}}_{T}}\right)\mathrm{erf}\left[\frac{\xi}{2}\left(\frac{D^{\mathrm{L}}_{T}}{D^{\mathrm{S}}_{T}}\right)^{1/2}\right] \qquad \text{(B2)}$$

where erf denotes the error function.

The boundary condition on the liquid–solid interface (Equation A1) with the aid of Equations B1, B2 and A4, after uncomplicated transformations becomes

$$h'_{\mathrm{L}}|_{\xi=\xi_{\mathrm{S}}} = \left\{\lambda_{\mathrm{S}}(1 - h_{\mathrm{SU}})/\lambda_{\mathrm{L}}\,\mathrm{erf}\left[\frac{\xi_{\mathrm{S}}}{2}\left(\frac{D^{\mathrm{L}}_{T}}{D^{\mathrm{S}}_{T}}\right)^{1/2}\right]\right\}$$

$$\times\left(\frac{D^{\mathrm{L}}_{T}}{\pi D^{\mathrm{S}}_{T}}\right)^{1/2} + \frac{L_{\mathrm{m}}\xi_{\mathrm{S}}}{2C^{\mathrm{L}}_{p}(T_{\mathrm{B}} - T_{\mathrm{S}})} \qquad \text{(B3)}$$

Here $h(\xi_{\mathrm{S}})$ and $h(0)$ are substituted with their equivalents. The substitution of Equation B2 into Equation 22 yields for the quenching rate into the volume of the crystalline phase the expression

$$|\dot{T}| = \frac{1}{2}\frac{V_{\mathrm{S}}}{x}(T_{\mathrm{B}} - T_{\mathrm{S}})\xi$$

$$\times\left(h_{\mathrm{SU}} + (1 - h_{\mathrm{SU}})\,\mathrm{erf}\left[\frac{\xi}{2}\left(\frac{D^{\mathrm{L}}_{T}}{D^{\mathrm{S}}_{T}}\right)^{1/2}\right]\right/$$

$$\times\left.\left\{\left(\frac{\pi D^{\mathrm{S}}_{T}}{D^{\mathrm{L}}_{T}}\right)^{1/2}\mathrm{erf}\left[\frac{\xi_{\mathrm{S}}}{2}\left(\frac{D^{\mathrm{L}}_{T}}{D^{\mathrm{S}}_{T}}\right)^{1/2}\right]\right\}\right) \qquad \text{(B4)}$$

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